

$D_{s1}^*(2710)$ and $D_{sJ}^*(2860)$ in the $\tilde{U}(12) \times O(3,1)$ -scheme

Tomohito Maeda*, Kenji Yamada*, Masuho Oda[†] and Shin Ishida^{1 **}

*Department of Engineering Science, Junior College Funabashi Campus, Nihon University,
Funabashi 274-8501, Japan

[†]School of Science and Engineering, Kokushikan University,
Tokyo 154-8515, Japan

**Research Institute of Science and Technology, College of Science and Technology, Nihon University,
Tokyo 101-8308, Japan

Abstract. In order to classify the charmed-strange mesons, including puzzling $D_{s0}^*(2317)$ and $D_{s1}(2460)$, and recently observed $D_{s1}^*(2710)$ and $D_{sJ}^*(2860)$, we employ the $\tilde{U}(12) \times O(3,1)$ level-classification scheme of hadrons proposed and developed by us in recent years. The scheme has a new degree of freedom, $SU(2)_\rho$, which leads to a number of extra states out of conventional $SU(6)$ -scheme, named *chiralons*. Applying this novel classification scheme, we investigate the strong decays of S - and P -wave $c\bar{s}$ mesons with one pseudoscalar emission by using the covariant oscillator quark model. As a result it is shown that $D_{s0}^*(2317)$ and $D_{s1}(2460)$ are described as $^1S_0^\chi$ and $^3S_1^\chi$ chiralons, forming the $SU(2)_\rho$ doublet with the ground-state D_s and D_s^* , respectively. Furthermore, the observed decay properties of $D_{s1}^*(2710)$ is consistently explained as the vector chiralon $^1P_1^\chi$. On the other hand, it is also found that the controversial narrow state, $D_{sJ}^*(2860)$, does not fit as predicted properties of our P -wave vector chiralon.

Keywords: $D_{s1}^*(2710)$, $D_{sJ}^*(2860)$, Strong Decays, Covariant Oscillator Quark Model, the $\tilde{U}(12) \times O(3,1)$ -scheme

PACS: 14.40.Lb, 13.25.Ft, 12.39.Ki

INTRODUCTION

In recent years we have proposed the $\tilde{U}(12) \times O(3,1)$ level-classification scheme [1] of hadrons, which corresponds to a covariant extension of the non-relativistic $SU(6) \times O(3)$ -scheme. In the new scheme, wave functions (WF) of composite hadrons are generally given by irreducible tensors of the $\tilde{U}(12) \times O(3,1)$. It is to be noted that, at the rest frame of hadrons, the representations of the $\tilde{U}(12)$ reduce to those of the $U(12)$. Hence the hadronic states can be classified by representations of the $U(12)$. The $U(12)$ includes a new degree of freedom $SU(2)_\rho$, called ρ -spin, in addition to conventional $SU(6) (\supset SU(2)_\sigma \times SU(3)_F)$, as $U(12) \supset SU(6) \times SU(2)_\rho$. An important feature of the scheme is that we include, apart from the $O(3,1)$ part, the ‘ ρ -spin down’ ($\rho_3 = -1$) component, which is treated as a fundamental building block² to construct the WF, effectively realizing only inside hadrons. Accordingly, we expect to exist a number of extra states out of the $SU(6)$ -framework, which is called chiralons.³

To check the validity of our new classification scheme, we investigate systematically the strong decays of $c\bar{s}$ mesons with one pseudoscalar emission by using the covariant oscillator quark model (COQM). Through the observed mass and results of decay study we present possible new assignments for observed charmed-strange mesons from the view point of the $\tilde{U}(12) \times O(3,1)$ -scheme.

FRAMEWORK OF THE COVARIANT OSCILLATOR QUARK MODEL

We briefly recapitulate a framework of the COQM⁴ relevant to the present application. The WF of a composite $s\bar{c}$ meson system is described by a bilocal field $\Phi(X, x)_\alpha^\beta$, (and its Pauli conjugate defined by $\bar{\Phi}(X, x)_\alpha^\beta =$

¹ Senior Research Fellow

² Physical meaning of $\rho_3 = -1$ component is discussed in the Ref. [2].

³ More strictly, chiralons are defined as the states which includes at least one $\rho_3 = -1$ component in the spin WF.

⁴ The COQM have been successfully applied to various (static and non-static) problem of hadrons for many years. As relatively recent application, for example, see Ref. [3] and references therein.

$(\gamma_4)_\alpha^{\alpha'} (\Phi^\dagger)_{\alpha'}^{\beta'} (\gamma_4)_\beta^{\beta'}$, where α and β represent Dirac spinor indices of s - and \bar{c} -quark, X and x denote the center of mass (CM) and relative coordinate four vectors, respectively.

We start from the Klein-Gordon type equation,

$$\left(\frac{\partial^2}{\partial X_\mu^2} - \mathcal{M}(x)^2 \right) \Phi(X, x)_\alpha^\beta = 0. \quad (1)$$

The squared-mass operator ⁵ (in the pure-confining harmonic oscillator (HO) potential limit) is given by

$$\mathcal{M}(x)^2 = d \left[-\frac{1}{2\mu} \frac{\partial^2}{\partial x_\mu^2} + \frac{K}{2} x_\mu^2 \right] \quad \left(d \equiv 2(m_s + m_{\bar{c}}), \quad \mu \equiv \frac{m_s m_{\bar{c}}}{m_s + m_{\bar{c}}} \right). \quad (2)$$

We can define the plane wave expansion concerning the CM motion

$$\Phi(X, x)_\alpha^\beta = \int \frac{d^3 \mathbf{P}}{\sqrt{(2\pi)^3 2P_0}} (e^{+iPX} \Phi(x, P)_\alpha^{(+)\beta} + e^{-iPX} \Phi(x, P)_\alpha^{(-)\beta}), \quad (3)$$

with respect to each level ($M_n = \sqrt{-P_\mu^2}$) determined by the squared-mass eigen-equation. In the above the positive / negative frequency parts $\Phi(x, P)^{(+/-)}$ denotes the internal WF of relevant ($s\bar{c}$) mesons with a definite HO mass. The complete set of (boosted) LS -coupling basis, generally represented by

$$\Phi(x, P)_\alpha^{(\pm)\beta} \sim f_{\mu_1 \mu_2 \dots}(v, x) \otimes \left(W(v)_\alpha^{(\pm)\beta} \right)_{\mu_1 \mu_2 \dots}, \quad (4)$$

is used to expand the internal WF, where $f(v, x)$ indicates the space-time part, while $W(v)_\alpha^{(\pm)\beta}$ does the spin part. Here $v_\mu = P_\mu/M$ is four velocity of meson. The concrete expressions of former part $f(v, x)$, being the eigen functions of \mathcal{M}^2 , are given by

$$f_G(v, x) = \frac{\beta}{\pi} \exp \left(-\frac{\beta}{2} (x_\mu^2 + 2(v_\mu x_\mu)^2) \right) \xrightarrow{\mathbf{v}=\mathbf{0}} \frac{\beta}{\pi} \exp \left(-\frac{\beta}{2} (\mathbf{x}^2 + \mathbf{x}_0^2) \right) \quad (\beta = \sqrt{\mu K}) \quad (5)$$

for S -wave ground states and

$$f_v(v, x) = a_v^\dagger f_G(x, v) = \frac{1}{\sqrt{2\beta}} (\beta x_v - \frac{\partial}{\partial x_v}) f_G(x, v) = \sqrt{2\beta} (x_v + v_v(x_p v_p)) f_G(x, v) \quad (6)$$

for P -wave excited states, respectively. On the other hand, the later part $W(v)^{(\pm)}$ consists of the direct product of respective Dirac spinor bases, which simulates the transformation properties of relevant constituent quarks, as

$$W_{r,r'}^{(+)}(v)_\alpha^\beta = u_r^{(s)}(v)_\alpha \bar{v}_{r'}^{(\bar{c})}(v)^\beta, \quad W_{r,r'}^{(-)}(v)_\alpha^\beta = v_r^{(s)}(v)_\alpha \bar{u}_{r'}^{(\bar{c})}(v)^\beta. \quad (7)$$

Here the index r represents the eigenvalue of ρ_3^6 in the rest frame ($\mathbf{v} = \mathbf{0}$). It should be noted that these spinors do not correspond to constituent quarks themselves. In fact, these contain only four velocity of hadron, hence the small component vanishes at the hadron rest frame.

⁵ By imposing the definite metric-type subsidiary condition to freeze a redundant relative-time degree of freedom, we get the eigen-values of $\mathcal{M}(x)^2$ as $M_n^2 = n\Omega + M_0^2$, where $\Omega = d\sqrt{\frac{K}{\mu}}$ and $n = L + 2N$ (N and L being the radial and orbital quantum numbers respectively), leading linear rising Regge trajectories.

⁶ For the anti-quark spinor $v_r(v)$, it should be understood as $\bar{\rho}_3 = -\rho_3^t$.

S- AND P-WAVE $s\bar{c}$ MESONS IN THE $\tilde{U}(12) \times O(3,1)$ -SCHEME

In this work we make the following assumptions for spin WF; only $\rho_3 = +1$ is allowed for c-quark, while both $\rho_3 = \pm 1$ can be realized for (rather) light s-quark.⁷ Resultant WF of S-wave states are given by

$$\begin{aligned}\Phi(x, P)_\alpha^{(+)\beta} &= f_G(v, x) \left[W_{+,+}^{(+)}(v)|_{(S=0)} + W_{+,+}^{(+)}(v)|_{(S=1)} + W_{-,+}^{(+)}(v)|_{(S=0)} + W_{-,+}^{(+)}(v)|_{(S=1)} \right]_\alpha^\beta \\ &= f_G(v, x) \frac{1}{2\sqrt{2}} \left[\left(i\gamma_5 \bar{D}_s(P) + i\gamma_\mu \bar{D}_{s\mu}^*(P) + \bar{D}_{s0}^\chi(P) + i\gamma_5 \gamma_\mu \bar{D}_{s1\mu}^\chi(P) \right) \left(1 + \frac{iP \cdot \gamma}{M_0} \right) \right]_\alpha^\beta,\end{aligned}\quad (8)$$

where $\{\bar{D}_s, \bar{D}_{s\mu}^*, \bar{D}_{s0}^\chi, \bar{D}_{s1\mu}^\chi\}$ represent local $s\bar{c}$ meson fields with $J^P = \{0^-, 1^-, 0^+, 1^+\}$, respectively. A superscript χ implies *chiralon*, s-quark being $\rho_3 = -1$. It should be noted that, in the relevant case, chiralons always have their ‘partners’ with opposite parity, same spin J , forming the ρ -spin doublet.⁸ Similarly, WF of P-wave states are given by

$$\begin{aligned}\Phi(x, P)_\alpha^{(+)\beta} &= f_v(v, x) \left[\left(W_{+,+}^{(+)}(v)|_{(S=0)} + W_{+,+}^{(+)}(v)|_{(S=1)} + W_{-,+}^{(+)}(v)|_{(S=0)} + W_{-,+}^{(+)}(v)|_{(S=1)} \right) \right]_\alpha^\beta \\ &= \sqrt{2\beta} x_v f_G(v, x) \frac{1}{2\sqrt{2}} \left[\left(i\gamma_5 \bar{D}_{s1v}(P) + i\gamma_\mu \bar{D}_{sJ\mu v}^*(P) + \bar{D}_{s1v}^\chi(P) + i\gamma_5 \gamma_\mu \bar{D}_{sJ\mu v}^{\chi*}(P) \right) \left(1 + \frac{iP \cdot \gamma}{M_1} \right) \right]_\alpha^\beta,\end{aligned}\quad (9)$$

where the local fields $\{\bar{D}_{s1v}, \bar{D}_{sJ\mu v}^*, \bar{D}_{s1v}^\chi, \bar{D}_{sJ\mu v}^{\chi*}\}$ correspond to $J^P = \{1^+, \{J = 0, 1, 2\}^+, 1^-, \{J = 0, 1, 2\}^-\}$ states, respectively⁹.

PIONIC / KAONIC DECAYS

Next we explain a procedure for calculating the pionic / kaonic decays of D_s mesons, applying the COQM. It can be considered that decays proceed through a single quark transition via emission of a local pion / kaon. We introduce the decay interactions as follows:

$$S_{\text{int}} = \int d^4x_1 \int d^4x_2 \langle \bar{\Phi}^{(-)}(x_1, x_2) V(x_1) \Phi^{(+)}(x_1, x_2) \rangle, \quad (10)$$

where x_1 and x_2 denote the space-time coordinates of s- and \bar{c} -quarks related to CM and relative coordinates as $X_\mu = (m_s x_{1\mu} + m_c x_{2\mu}) / (m_s + m_c)$, $x_\mu = x_{1\mu} - x_{2\mu}$, and $\langle \cdots \rangle$ means taking trace concerning flavor and Dirac indices. Two types of vertex factors, $V(x_1) = V_{ND}(x_1) + V_D(x_1)$, denoting non-derivative and derivative couplings¹⁰, are

$$\langle \bar{\Phi}^{(-)}(x_1, x_2) V_{ND}(x_1) \Phi^{(+)}(x_1, x_2) \rangle = dg_{ND} \langle \bar{\Phi}^{(-)}(x_1, x_2) (i\gamma_5 \phi_{ps}(x_1)) \Phi^{(+)}(x_1, x_2) \rangle, \quad (11)$$

$$\langle \bar{\Phi}^{(-)}(x_1, x_2) V_D(x_1) \Phi^{(+)}(x_1, x_2) \rangle = \frac{dg_D}{2m_s} \langle \bar{\Phi}^{(-)}(x_1, x_2) \left(\vec{\partial}_{x_{1\mu}} \phi_{ps}(x_1) \right) \left(\gamma_5 \sigma_{\mu\nu} (\vec{\partial}_{x_1} - \overleftarrow{\partial}_{x_1}) \right) \Phi^{(+)}(x_1, x_2) \rangle. \quad (12)$$

Rewriting the above with CM and relative coordinates by

$$\Phi^{(+)}(x_1, x_2) \sim \Phi^{(+)}(x, P) e^{+iP \cdot X}, \quad \bar{\Phi}^{(-)}(x_1, x_2) \sim \bar{\Phi}^{(-)}(x, P) e^{-iP' \cdot X} \quad (13)$$

⁷ In the Ref. [2], $\rho_3 = -1$ component for c-quark is taken into account.

⁸ Clearly degeneracy of mass (in the HO limit) for the $SU(2)_\rho$ doublets is badly broken. Thus the $SU(2)_\rho$ should be considered, in contrast to the $SU(2)_\sigma$ -symmetry, just to offer a tool which gives a new perspective on classifying hadronic states.

⁹ All Lorentz indices of local fields satisfy the subsidiary conditions; $v_\mu D_{s\mu}^* = v_\mu D_{s1\mu}^\chi = v_\mu D_{s1\mu} = v_\mu D_{sJ\mu v}^* = v_\mu D_{s1\mu}^\chi = v_\mu D_{sJ\mu v}^* = 0$, $D_{s0, 2\mu v}^* = D_{s0, 2v\mu}^*$, $D_{s1\mu v}^* = -D_{s1v\mu}^*$, and $D_{s2\mu\mu}^* = 0$.

¹⁰ In the non-relativistic limit, the first term contributes only the transitions between chiralons and non-chiralons, accompanied by ρ_3 -change. On the other hand, the second term contributes only transitions among non-chiralons or chiralons themselves, which gives the well known $\sigma \cdot (\mathbf{q} - \frac{q_0}{m_q} \mathbf{p}_q)$ vertex.

TABLE 1. Possible assignments of S - and P - wave D_s mesons in the $\tilde{U}(12)_{SF} \times O(3,1)$ -scheme

n	L	P	V	S^χ	A^χ
0	0	$1^1 S_0$ 0^-	$1^3 S_1$ 1^-	$1^1 S_0^\chi$ 0^+	$1^3 S_1^\chi$ 1^+
		$D_s(1968)$	(2.11 GeV) $D_s^*(2112)$	$D_{s0}^*(2317)$	(2.46 GeV) $D_{s1}(2460)$
1	1	$1^1 P_1$ 1^+	$1^3 P_{J=0,1,2}$ $\{0, 1, 2\}^+$	$1^1 P_1^\chi$ 1^-	$1^3 P_{J=0,1,2}^\chi$ $\{0, 1, 2\}^-$
		$D_{s1}(2536)$	(2.57 GeV) $D_{s0}^*(\sim 2573), D_{s1}(\sim 2573), D_{s2}(2573)$	$D_{s1}^*(2710)$	(2.87 GeV) $D_{s0}(\sim 2866), D_{sJ}^*(2860)?, D_{s2}(\sim 2866)$

TABLE 2. Results on pionic / kaonic transition widths (in MeV)

Mesons	$^{2S+1}L_J; J_q^P$	$(\rho_3(c), \bar{\rho}_3(\bar{s}))$	decay channel	$\Gamma^{\text{theor.}}$	$\Gamma^{\text{exp.}}$
D_s	$1^1 S_0; \frac{1}{2}^-$	$(+, +)$	-	-	-
D_s^*	$3^1 S_1; \frac{1}{2}^-$	$(+, +)$	$D_s \pi^0$	0.0020 keV	<110 keV
$D_{s0}^*(2317)$	$1^1 S_0^\chi; \frac{1}{2}^+$	$(+, -)$	$D_s \pi^0$	9.2 keV	<3.8 MeV
$D_{s1}^*(2460)$	$3^1 S_1^\chi; \frac{1}{2}^+$	$(+, -)$	$D_s^* \pi^0$	8.2 keV	< 3.5 MeV
$D_{s0}(\sim 2573)^*$	$3^1 P_0; \frac{1}{2}^+$	$(+, +)$	$D K$	$\sim 184 \text{ MeV}$	
$D_{s1}(\sim 2573)$	$P_1; \frac{1}{2}^+$	$(+, +)$	$D^* K$	$\sim 184 \text{ MeV}$	
$D_{s1}(2536)$	$P_1; \frac{3}{2}^+$	$(+, +)$	$D^* K$	0.22 MeV	< 2.3 MeV
$D_{s2}^*(2573)$	$3^1 P_2; \frac{3}{2}^+$	$(+, +)$	$D K + D^* K$	$18.9+0.96=19.9 \text{ MeV}$	$20 \pm 5 \text{ MeV}$
$D_{s1}^*(2710)$	$1^1 P_1^\chi$	$(+, -)$	$D K + D^* K$	$63 + 43 = 106 \text{ MeV}$	$149_{-59}^{+46} \text{ MeV}$
$D_{sJ}^*(\sim 2860)$	$3^1 P_1^\chi$	$(+, -)$	$D K + D^* K$	$177+60=237 \text{ MeV}$	$48 \pm 9 \text{ MeV}$

and

$$\phi_{\text{ps}}(x_1) \sim \phi_{\text{ps}}(q) e^{-iq \cdot x_1} = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta^{(8)}}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta^{(8)}}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}} \eta^{(8)} \end{pmatrix} e^{-iq \cdot (\frac{2mc}{d}x + X)} \quad (q_\mu = P_\mu - P'_\mu), \quad (14)$$

we obtain a formula to calculate the decay amplitudes as

$$T = dg_{ND} \int d^4x \langle \bar{\Phi}^{(-)}(P', x) i \gamma_5 \phi_{\text{ps}}(q) \Phi^{(+)}(P, x) \rangle e^{-i\frac{2mc}{d}q \cdot x} \quad (15)$$

$$+ g_D \int d^4x \langle \bar{\Phi}^{(-)}(P', x) \gamma_5 q_\mu \sigma_{\mu\nu} \left(P_\nu + P'_\nu - \frac{d}{2m_s} i (\vec{\partial}_{x,\nu} - \overleftarrow{\partial}_{x,\nu}) \right) \phi_{\text{ps}}(q) \Phi^{(+)}(P, x) \rangle e^{-i\frac{2mc}{d}q \cdot x}. \quad (16)$$

ASSIGNMENTS AND NUMERICAL RESULTS

New charmed strange meson $D_{sJ}^*(2860)$ was first observed by the BaBar collaboration[7] in the DK channel of e^+e^- inclusive measurement with $M = 2856.6 \pm 1.5 \pm 5.0 \text{ MeV}$ and $\Gamma = 48 \pm 7 \pm 10 \text{ MeV}$. Shortly after, a $J^P = 1^-$ state $D_{s1}^*(2710)$ was reported by the Belle collaboration[9] with $M = 2708 \pm 9_{-10}^{+11} \text{ MeV}$ and $\Gamma = 108 \pm 23_{-31}^{+36} \text{ MeV}$ in the DK invariant mass distribution of B -decay. In the latest report from the BaBar collaboration[8], the $D_{sJ}^*(2860)$ and $D_{s1}^*(2710)$ were seen in both DK and D^*K decay modes with the ratios of branching fraction

$$\frac{BR(D_{s1}^*(2710) \rightarrow D^*K)}{BR(D_{s1}^*(2710) \rightarrow DK)} = 0.91 \pm 0.13 \pm 0.12, \quad \frac{BR(D_{sJ}^*(2860) \rightarrow D^*K)}{BR(D_{sJ}^*(2860) \rightarrow DK)} = 1.10 \pm 0.15 \pm 0.19. \quad (17)$$

To understand the nature of these newly observed states, many theoretical efforts have been done,¹¹ but it is still a subject of controversy.

Now we discuss a classification based on the $\tilde{U}(12) \times O(3,1)$ -scheme, shown in Table 1. Subsequent results for pionic / kaonic transition widths in comparison with experiments are also given in Table 2. In both tables, some predicted, but experimentally missing states are underlined.¹² In consideration of unexpectedly lower mass, D_{s0}^* (2317) and D_{s1} (2460) are plausible candidates for our S -wave chiralons. Thus we assign them to D_{s0}^χ and D_{s1}^χ chiralons in Eq. (8), respectively. A issue raised by relevant assignments is whether there are additional (conventional) $c\bar{s}$ P -wave states, D_{s1}^* and D_{s1}^{*13} in Eq. (9). The mass of those P -wave 1^+ non-chiralons are expected to be about ~ 2.57 GeV from that of typical D_{s2}^* (2573), being much heavier than S -wave chiralons. The results of strong decays, assuming the existences of D_{s0}^* (~ 2.57) and $D_{s1}^{j=1/2}$ (~ 2.57) non-chiralons in Table 2 indicate that it is not contradict with present experiment, owing to predicted large widths. On the other hand, the experimental known state D_{s1} (2536) is naturally explained as $D_{s1}^{j=3/2}$ non-chiralons, being mixing partner of $D_{s1}^{j=1/2}$. Next we make a rough estimate the mass of the P -wave chiralons, by using global HO mass relation, $M_1^2 = \Omega + M_0^2$, derived from Eq. (2). As a result, we predict two vector chiralons, D_{s1}^χ and $D_{s1}^{*\chi}$, with the mass about $2.7 \sim 2.9$ GeV. These states are possible candidate of recently reported D_{s1}^* (2710) and D_{sJ}^* (2860). We calculate the strong decays to check this possibility, and found that D_{s1}^* (2710) meson is consistently explained as the vector chiralon $^1P_1^\chi$. On the other hand, D_{sJ}^* (2860), does not fit as predicted properties of our P -wave vector chiralon.

CONCLUDING REMARKS

In conclusion, the D_{s0}^* (2317), D_{s1} (2460), and D_{s1}^* (2710) are good candidates for $c\bar{s}$ chiralons through their observed masses and decay properties. The existence of P -wave non-chiralon D_{s0}^* and D_{s1} with broad width (~ 180 MeV) and higher mass (~ 2.57 GeV) appeared in the DK and D^*K spectrum, respectively, and that of $J^P = \{0, 1, 2\}^-$ P -wave chiralons with the mass $2.7 \sim 2.9$ GeV should be checked in future experiment.

ACKNOWLEDGMENTS

This work was supported in part by Nihon University Research Grant for 2008.

REFERENCES

1. S. Ishida, M. Ishida and T. Maeda, *Prog. Theor. Phys.* **104** (2000) 785; S. Ishida and M. Ishida, *Phys. Lett.* **B539** (2002) 249.
2. K. Yamada, in these proceedings.
3. M. Oda, K. Nishimura, M. Ishida, and S. Ishida, *Prog. Theor. Phys.* **133** (2000) 1213.
4. C. Amsler, et al., (Particle Data Group), *Phys. Lett.* **B667** (2008) 1.
5. T. Matsuki, T. Morii, and K. Sudoh, *Eur. Phys. J.* **A31** (2007) 701.
6. F.E. Close, C.E. Thomasa, O. Lakhinab and E. S. Swanson, *Phys. Lett.* **B647** (2007) 159.
7. B. Aubert et al., BABAR Collaboration, *Phys. Rev. Lett.* **97** (2006) 222001.
8. B. Aubert, et al., BaBar Collaboration, *Phys. Rev.* **D80** (2009) 092003.
9. K. Abe, et al., Belle Collaboration, hep-ex/0608031; J. Brodzicka et al., Belle Collaboration, *Phys. Rev. Lett.* **100** (2008) 092001.
10. P. L. Cho and M. B. Wise, *Phys. Rev.* **D49** (1994) 6228.

¹¹ For example, the preceding work[5, 6] predicts D_{sJ}^* (2860) and D_{s1}^* (2710) as $2P$ and $2S$ (or $1D$) states of conventional $c\bar{s}$ states.

¹² Though the decay studies we have used following parameters:

(i) coupling constants; $g_D = g_A/\sqrt{2}f_\pi(K)$, $g_A = 0.55$, $g_{ND} = 8.0$ GeV. ($f_\pi(K) = 94$ (114) MeV) (ii) HO ($SU(2)_\rho$ -symmetric) mass; $M_0 = 2.26$ GeV, $M_1 = 2.69$ GeV. (iii) Regge slope inverse; $\Omega = 2.160$ GeV², determined from $M(D_{s2}^*(2573))^2 - M(D_s^*(2112))^2$. (iv) quark mass ratio; $m_c/m_s = 1.55/0.45$. (v) $\eta^{(8)} - \pi_0$ mixing angle; $\sin^2 \theta = 0.65 \times 10^{-4}$ [10].

¹³ More properly, mixed states $|D_{s1}^{j=1/2}\rangle = \sqrt{2/3}|D_{s1}^*\rangle - \sqrt{1/3}|D_{s1}\rangle$ and $|D_{s1}^{j=3/2}\rangle = \sqrt{1/3}|D_{s1}^*\rangle + \sqrt{2/3}|D_{s1}\rangle$ are realized in the heavy quark limit.